Reactor Analysis with First-Order Reaction Kinetics

CEEG 340–Introduction to Environmental Engineering
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Continuous Input–CMFR vs. PFR–First Order Reaction

The ideal CMFR and PFR are fundamentally different, and, thus perform differently. When a parcel of fluid enters a CMFR, it is immediately mixed throughout the reactor. In contrast, when a parcel of fluid enters a PFR it remains separate as it moves through the reactor.

The goal of following problems is to illustrate how this difference affects the performances of CMFRs and PFRs at steady state, as the choice of reactor type is an important design consideration.

You have been asked to choose between a a well mixed treatment pond (behaves like a CMFR) and a constructed wetland (behaves like a PFR) to remove nitrogen from agricultural runoff. Assume that nitrogen is removed biologically at a rate that is first order.

The influent concentration of nitrogen, C_{in}; the flowrate, Q; and the first-order reaction rate constant, k, are known and are the same for both reactors. Q = 5 m^3/day; k = 0.1 day^{-1}.
If the steady-state effluent concentration, C_{out}, equals half of C_{in} ($C_{out} = 0.5 \times C_{in}$), what volume of reactor is required for a

1. PFR?

$C_{out} = C_{in} e^{-k\theta}$

$0.5 C_{in} = C_{in} e^{-k\theta}$

$\ln 0.5 = -0.1\theta$

$\theta = 6.9$ DAY

$\theta = \frac{V}{Q}$

$V = \theta \times Q =$

$V = 35 m^3$

2. CMFR?

$C = \frac{C_{in}}{1+k\theta}$

$0.5 C_{in} = \frac{C_{in}}{1+k\theta}$

$0.5 = \frac{1}{1+k\theta}$

$2 = 1 + k\theta$

$\theta = \frac{1}{k} = 10$ DAYS

$V = \theta \times Q = 10_{DAYS} \times 5\ m^3/_{DAY}$

$V = 50 m^3$

FOR 1st ORDER:

$V_{PFR} < V_{CMFR}$

CMFR–The Monster Equation

A small well-mixed pond ($V = 500\ m^3$) is contaminated with polyfluoroalkyl substances (PFAS), concentraton = 0.01 μgram/liter. In addition a stream that enters and exits the pond ($Q = 50\ \frac{m^3}{day}$) is contaminated with 10 μg/L of PFAS due to contamination of flame retardants. PFAS biodegrades very slowly, and has a reaction rate coefficient, $k = 0.003\ day^{-1}$.

1. What is the concentration of PFAs after 3 days?

$$V \frac{dC}{dt} = QC_{in} - QC_{out} - rV$$

$$V \frac{dC}{dt} = QC_{in} - QC - kCV$$

$$\theta = \frac{V}{Q} = \frac{500\ m^3}{50\ m^3/day} = 10\ DAY$$

INTEGRATE

$$C = \frac{C_{in}}{1+k\theta}\left\{1 - \exp\left[-\left(\frac{1}{\theta}+k\right)t\right]\right\} + C(0)\exp\left[-\left(\frac{1}{\theta}+k\right)t\right]$$

2.58   0.2658

$$C = \frac{10\ \mu g}{1 + 0.003\frac{1}{day} \times 10\ day}\left\{1 - \exp\left[-\left(\frac{1}{10}+0.003\right)\times 3\right]\right\} +$$

$$+\ 0.01 \times \exp\left[-\left(\frac{1}{10}+0.003\right)\times 3\right]$$

0.0073

C = 2.6 μg/L AFTER 3 DAYS